STUDIES IN THE HISTORY OF PROBABILITY AND STATISTICS*

I. DICING AND GAMING (A NOTE ON THE HISTORY OF PROBABILITY)

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'See, this is new? It hath been already of old time.' (Ecclesiastes i. 10.)

1. A cynical archaeologist remarked recently that a symptom of decadence in a civilization is when men become interested in their own history, and he added that in the unlikely eventuality of any proof being required of the decadence of this phase of *Homo sapiens* it could be found in the present-day interest in archaeology. Most generalizations of a sweeping character such as this are unacceptable, chiefly because there is no way of putting them to proof; but the present interest of scientists in general, and of statisticians in particular, in the origins of scientific thought, is far from implying the decadence of science, whatever may be implied by an interest in the arts.

It is inviting, and at the same time profitless, to speculate why modern scientists have such an interest. The possibility of deciding priority of discovery which concerned the Victorian scientist so closely does not cause much controversy to-day, for the modern scientist would hold that to ascribe any discovery in the field of science to any single person is unrealistic. Thus, while we are taught at school, for example, that Newton and Leibnitz separately and independently 'discovered' the differential calculus, it would perhaps be more appropriate to say that Newton and Leibnitz each supplied the last link in the chain of reasoning which gave us the differential calculus—a chain which can be traced back through Pierre Fermat, Barrow, Torricelli and Galileo, and that it is surprising that there were only two mathematicians who did this.

Mathematics is essentially an expression of thought in which we build on the mental effort of our forerunners, and probability is no exception to this general rule. The real difficulty we meet with in trying to trace probability back to its origins is that it started essentially as an empirical science and developed only lately on the mathematical side. It is hard to say where in time the change came from empiricism to mathematical formalism as it appears to have taken place over hundreds of years; and the claims put forward for Pascal and Fermat as the creators of probability theory cannot entirely be substantiated.

2. When man first started to play games of chance is a time problem we shall never clearly resolve. We may place on record that it is a commonplace thing for archaeologists to find a preponderance of astragali† among the bones of animals dug up on prehistoric sites. One archaeologist stated that he had found up to seven times as many as any other bone, another put the figure at 500 (sic!), while yet a third, refusing to be drawn to a figure, stated that they were many. This fact has probably little significance. The astragalus has little marrow in it and was possibly not worth cracking for the sake of its contents as were the long bones; it is knobbly and presents no flat curves for drawing as does the shoulder

* [*Editorial note*. It is hoped to publish articles by a number of different authors under this general heading.]

† The *astragalus* is a small bone in the ankle, immediately under the *talus* or heel-bone. See Pl. 2*a*.

blade for example. All we may do is to place on record that round about 40,000 years ago there were large numbers of the astragali of sheep, goat and deer lying about.

The astragali of animals with hooves are different from those with feet such as man, dog and cat. From the comparison in Text-fig. 1 we note how in the case of the dog the astragalus is developed on one side to allow for the support of the bones of the feet. The astragalus of the hooved animal is almost symmetrical about a longitudinal axis and it is a pleasant toy to play with. In France and Greece children still play games with them in the streets, and it is possible to buy pieces of metal fashioned into an idealized shape but still recognizable as astragali.

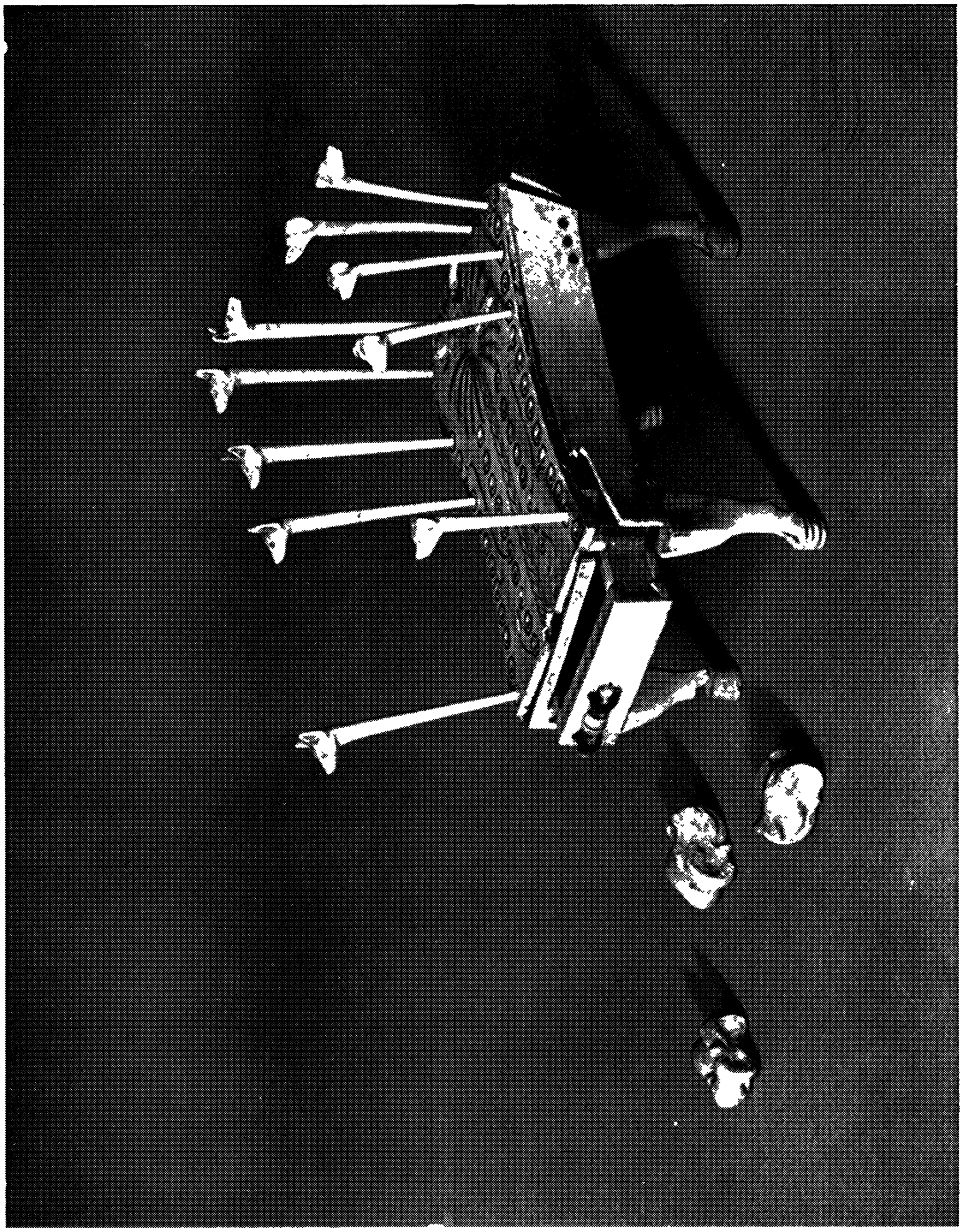
Text-fig. 1. Drawings of the astragalus in sheep and dog, natural size.

3. Some time between prehistoric man of four hundred centuries ago and the beginning of the third millennium before Christ *Homo sapiens* invented games and among these games, games of chance. We know from paintings, terra-cotta groups, etc., that the astragalus was used in Greece like the ancient quoit,* but there is no doubt from paintings on tombs in Egypt and excavated material that the use of the astragalus in games where it is desired to move counters was well established by the time of the First Dynasty. In one painting a nobleman, shown playing a game in his after-life, delicately poises an astragalus on his finger tip, a board with 'men' in front of him. A typical game of *c.* 1800 B.C. is that of 'Hounds and Jackals' illustrated in Pl. 1. The game seems similar to our present-day 'Snakes and Ladders'; the hounds and jackals were moved according to some rule by throwing the astragali found with the game and shown in the figure. Variants of this game were undoubtedly played from the time of the First Dynasty (*c.* 3500 B.C.).

It is possible but not altogether likely that these games originated in Egypt. They certainly did not originate in Greece, as has been claimed for reasons which we shall give later. However, Herodotus, the first Greek historian, like his present-day counterparts, was willing to believe that the Greeks (or allied peoples) had invented nearly everything. His claim that the Lydians introduced coinage has about as much foundation as his claim regarding games of chance. He writes (*c.* 500 B.C.) about the famine in Lydia (which was *c.* 1500 B.C.) as follows:

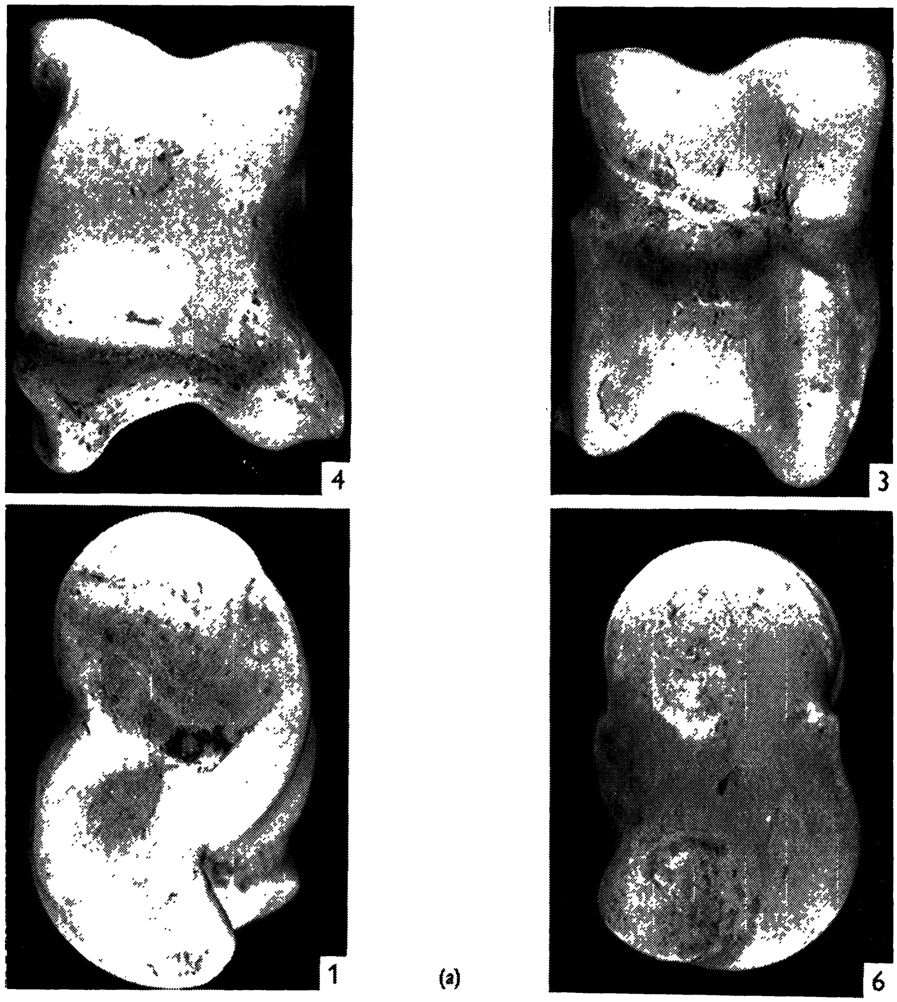
> The Lydians have very nearly the same customs as the Greeks. They were the first nation to introduce the use of gold and silver coins and the first to sell goods by retail. They claim also the invention of all games which are common to them with the Greeks. These they declare that they invented about the time that they colonized Tyrrhenia, an event of which they give the following account. In the days of Atys, the son of Manes, there was great scarcity through the whole land of Lydia. For some time the Lydians bore the affliction patiently, but finding that it did not pass away,

* From the name 'knucklebone' we might infer that among the early games were those in which the astragali were balanced on the bones of the knuckles and then tossed and caught again.



The game of Hounds and Jackals.

(*Facing p.* 2)



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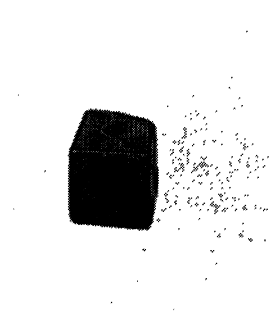


(c)

they set to work to devise remedies for the evil. Various expedients were discovered by various persons; dice and huckle-bones (i.e. astragali) and ball and all such games were invented, except tables,* the invention of which they do not claim as theirs. The plan adopted against famine was to engage in games on one day so entirely as not to feel any craving for food, and the next day to eat and abstain from games. In this way they passed eighteen years.

In yet another commentary we are told that games of chance were invented during the Trojan war by Palamedes. During the 10-year investment of the city of Troy various games were invented to prevent the soldiers' morale suffering from boredom.

4. The game of ball is mentioned by Homer and according to Plato was evolved in Egypt. It is not, however, a game of chance. The story of dice we shall return to, but we may first carry the story of the astragalus a little further. In the early part of the first millennium it would seem that astragali were used by both adults and children for their leisure games. Homer (*c.* 900 B.C.) tells us that when Patroclus was a small boy he became so angry with his opponent while playing a game of knucklebones that he nearly killed him. Another writer of the same period tells us that students played knucklebones everywhere, that they were acclaimed as presents and that as a prize for handwriting one student was given eighty astragali all at once! It is not difficult to imagine the small boys of that era collecting astragali as they collected marbles, much as the boys of our own era still do.

That the astragalus was used commonly in the gaming which the Greeks and later the Romans conducted with zeal and passion, the references in the literature of that time leave no room for doubt. One of the chief games may have been the simple one of throwing four astragali together and noting which sides fell uppermost. The astragalus has only four sides on which it may rest, and it has been deduced, among others by Nicolas Leonicus Thomeus (1456–1531), that a common method of enumeration was that the upper side, broad and slightly convex counted 4, the lower side broad and hollowed 3, the lateral side narrow and flat 1 and the other lateral which is slightly hollow 6. These aspects of a sheep's astragalus are shown in Pl. 2*a*. (With present-day astragali the probabilities of scoring 1 and 6 are each approximately equal to 1/10 and those of 3 and 4 approximately 4/10.) The worst throw for the Greeks with one bone was unity which they called the dog, and sometimes the vulture. The best of all throws with four knucklebones was the throw of Venus when all four sides were different which has an actual probability of about 1/26. But at different times and in different games the numbers must have been varied, for the throw of Euripides with four astragali, discussed by several fifteenth-century writers, was worth 40. How the bones fell to achieve this result is not stated, although Cardano writing in the sixteenth-century states that it was four fours. (Probability *c.* 1/39.)

5. In classical Rome the astragalus was imitated in carved stone with figures and scenes incised on the sides. A typical example is illustrated in Pl. 2*b*. Stone astragali have also been found in Egypt. At this time too we have the production of lewd figures in metal or bone varying in size from about 1 cm. to over 1 in. in height. That these figures were used for gaming may be deduced from the fact that the six possible positions in which the figure may fall are each marked with a number of dots.†

Besides the astragali it appears possible that throwing sticks were also used for games of chance, although it may be that they had a greater religious significance; we shall return

* This may have been an early form of backgammon or may have been shuffle-board.

† I have not tested these figures for bias. They are a development, I think, of dice rather than astragali.

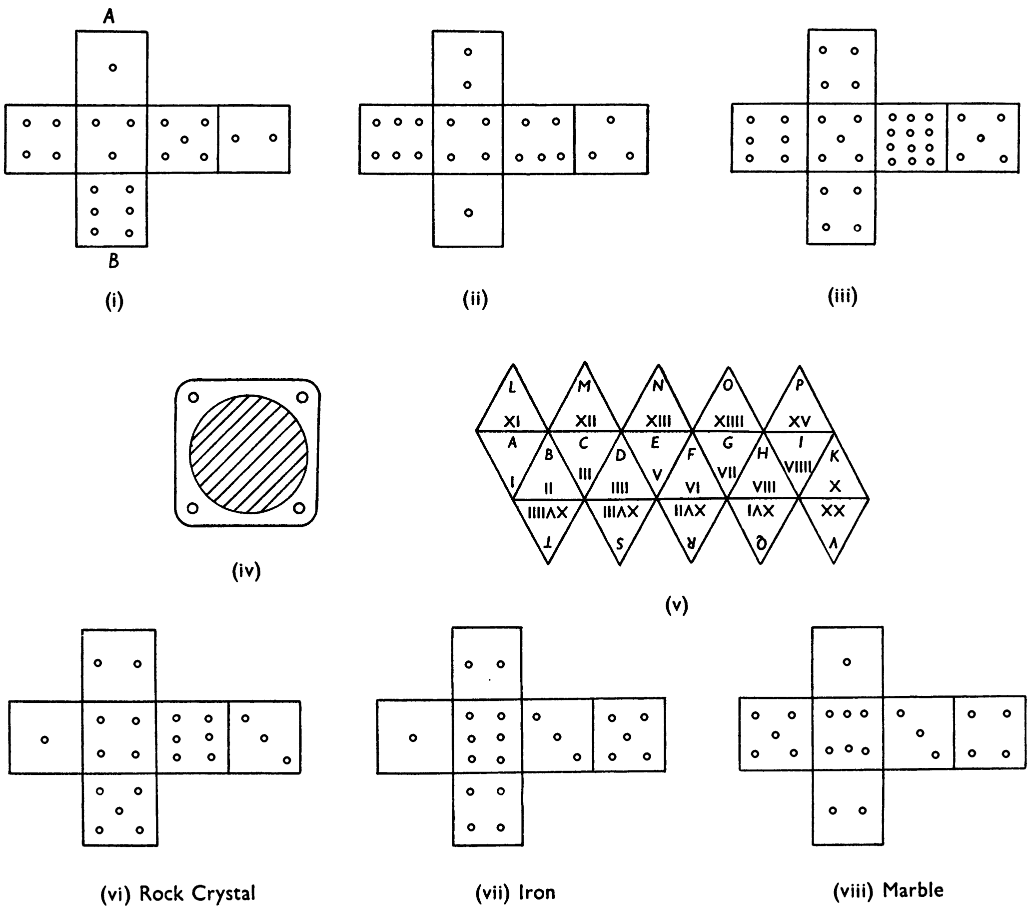
to this point later. The throwing stick was made of wood or ivory and was often approximately 3 in. in length with cross-section when square of about 1 cm. each way. Such throwing sticks were known to the ancient Britons, to the Greeks, the Romans, the Egyptians and to the Maya Indians of the American continent. Sometimes the sticks are elliptical in cross-section with major axis of approximately 1 cm., but they are all alike in having only four numbers on them, one at each end of the upper face and one at each end of the lower parallel face. In the European throwing sticks the majority of numbers are marked by small engraved circles, but they are sometimes indicated by cuts in the wood or ivory. The Maya throwing sticks are marked by coloured scratches in ivory. The actual numbers marked vary. They are mostly 1, 2, 5, 6, but 3 and 4 have also been noticed. These throwing sticks are of little importance in gaming. They are mentioned because it is interesting to note the likelihood that gaming originated at many points, and, although this is a remark one could not defend, that it possibly was originally a debasement of a religious ceremony.

6. The six-sided die may have been obtained from the astragalus by grinding it down until it formed a rough cube. The Musée de Louvre has several astragali which have been treated in this way, but one cannot imagine they formed very satisfactory dice. The honeycombed (or cancellous) bone tissue has been exposed in several places and the crude die would clearly not have a long life. Whether the die was evolved in this way or not the evolution must have taken place some considerable time before Christ. The earliest die known was excavated in northern Iraq and is dated at the beginning of the third millennium. It is described as being of well-fired buff pottery. The dots are arranged as shown in Text-fig. 2(i), the edges at A and B being imagined folded away from the reader. It will be noted that the opposite points are in consecutive order, 2 opposite 3, 4 opposite 5 and 6 opposite 1.

A die excavated in Mohenjo-Daro (Ancient India) is also dated as third millennium, and it is also made of hard buff pottery. The order of the points is again consecutive, but this time we have 1 opposite 2, 3 opposite 4, 5 opposite 6. Few other dice have been recorded in this millennium. At the time of the XVIIIth dynasty in Egypt (*c.* 1400 B.C.) a die with the markings shown in Text-fig. 2 (ii) must have been in play. The arrangement of the five dots is unusual. Somewhere about this time, however, the arrangement of the numbers settled down to the familiar two-partitions of 7 opposite one another as shown in Text-fig. 2 (vi), which arrangement has persisted until to-day. Out of records, collected by the present writer, of some fifty dice of classical times made of crystal, ivory, sandstone, ironstone, wood and other materials, forty had the two-partition of 7 arrangement. A twelfth-century (A.D.) Greek bishop wrote that this was the way in which a die should be marked, and a sixteenth-century gambler theorizes that this arrangement was chosen to make it easy to check whether all the numbers had been marked on the die and no figure duplicated at the expense of leaving out another. One die of the first millennium is said to have 9 opposite 6, 5 opposite 3, and 4 opposite 2. It may have been especially made for a particular game; alternatively, it is possible that it may have some ceremonial significance. This is possibly also true of a die marked as in Text-fig. 2 (iii), although it might have been a die used for cheating.

7. That dice were used in Egypt is clear from the XVIIIth-dynasty specimen. It is thought, however, that dicing did not become common until the advent of the Ptolemaic dynasty (300 to 30 B.C.) which originated from Greece. Several dice are known of this

period including a beautiful specimen in hard brown limestone of side *c.* 1 in., which has the sacred symbols of Osiris, Horus, Isis, Nebhat, Hathor and Horhudet engraved on the six sides. This would almost undoubtedly have been used for some form of divination rites.



Text-fig. 2

Dice in Britain were in a very primitive state at the time of Christ. The pieces used then were formed by roughly squaring the long bone of an animal and cutting it into sections to form objects approximately cubical in shape. The marrow was taken out leaving a hollow square cylinder of which the cross-section in diagrammatic form is sketched in Text-fig. 2 (iv). (The British Museum has two of these.) These primitive dice had the two partition of 7 arrangement, 3 being opposite 4 on the hollow ends. Some dice had a 3 on each end and no 4. Several dice of this kind have been excavated in the chalk and flint country and dated late in the first millennium.

The working out of the geometry of solid figures by Greek mathematicians appears to have been followed almost immediately by the construction of polyhedral dice. A beautiful icosahedron in rock crystal now at the Musée de Louvre is the most famous of these. (In the diagram of Text-fig. 2(v) it may be imagined the outline is folded away from the reader.) A figure with 19 faces badly cut but apparently imagined to be rectangular has a roman digit on each face from I to X, and above that the numbers rise by tens to C. The number LXXX is missing, but the number XX appears twice on one face. There was also a die of 18 faces probably formed by beating out a cubical die, a die with 14 faces and so on.

Faked dice were also not unknown. Apart from the device of leaving out one number and duplicating another it is stated that hollow dice have been found dating from Roman times. The unity on the face of the die forms a small round plate which can be lifted. It is suggested that a small ball of leather could be crammed into the hollow of the die through this hole in such a way as to cause the die to tend to fall in a predetermined manner.

8. Gaming reached such popularity with the Romans that it was found necessary to promulgate laws forbidding it except at certain seasons. What game was played by the common people we do not know, but there are many references to those played by the emperors. In Suetonius's *Life of Augustus* (Loeb's translation) we find:

> He (Augustus) did not in the least shrink from a reputation for gaming and played frankly and openly for recreation, even when he was well on in years, not only in the month of December, but on other holidays as well and on working days too. There is no question about this, for in a letter in his own handwriting he says, 'I dined, dear Tiberius, with the same company; ... We gambled like old men during the meal both yesterday and to-day, for when the dice were thrown whoever turned up the "dog" or the 6, put a denarius in the pool for each one of the dice, and the whole was taken by anyone who threw the Venus'.

There are several other references to gaming in this *Life*. Whether the word talis should be translated as dice or astragali (knucklebones) is a moot point. The die as we know it is usually referred to as 'tessera'. The astragalus is often called the talus (or heel-bone), and this is the word Suetonius actually used. From the description of the play it would seem appropriate to read knucklebones for dice.

In Suetonius's *Life of Claudius* we are told that Claudius was so devoted to dicing that he wrote a book about it, and that he used to play while driving, throwing on to a board fitted especially in his carriage. From another source we learn that he played right hand against left hand.

9. These two instances are chosen to illustrate the passion for gaming which apparently possessed the Romans, and it is possible to cite many others. The question which constantly recurs to one while studying these games of the past is 'Why did not someone notice the equi-proportionality property of the fall of the die?' It is understandable that no theory was made to describe the fall of the astragalus. But the Greeks had performed the necessary abstraction of thought to make the mathematical idealization of the cube (and other solid figures); at first sight it seems curious that mathematicians did not then go on a little further and give equal weight to each side of the cube and so on. For if dicing and gaming generally were carried on by so many persons for so long that it was thought necessary to prohibit them, surely someone must have noticed that with a cube on the average any one side turned up as frequently as any other? We can only make guesses on this point, but it would seem to the writer that there are two possible explanations, the imperfections of the dice and their use in religious ceremonies.

10. *Imperfect dice.* We speak of a true or a fair die nowadays when we mean that there is no bias apparent when the die is thrown. In Roman times, and presumably earlier, it seems to have been the exception rather than the rule for the die to be true. Many dice of the classical period have been thrown by the writer and they were nearly all biased but not all in the same way. For example, three classical dice from the British Museum gave the results shown in the table from 204 throws each. The arrangement of the pips on the dice

were as in Text-fig. 2(vi), (vii) and (viii). The rock crystal is a beautifully made die; the others are a little primitive, and the sides of the iron die are only approximately parallel. The marble and the iron dice are obviously biased, and this was true of many of the other dice examined. A photograph of a wooden die of the classical period is given in Pl. 2(*c*). It will be noted that one of the faces shown is not square, and the impression one has is that the owner picked up a piece of wood of convenient shape, smoothed it a little and engraved the pips. It would therefore have been difficult, except over a long period, to notice any regularity.

Number of pips ...	1	2	3	4	5	6
Rock crystal	30	38	31	34	34	37
Iron	35	39	30	21	37	42
Marble	27	28	23	47	25	54

11. *Divination.* In spite of the imperfections of the dice it is probable that some theory might have been made if magic or religion or both had not been involved also. A scheme whereby the deity consulted is given an opportunity of expressing his wishes appears to be a fundamental in the development of all religions. As late as 1737 we have John Wesley deciding by the drawing of lots whether to marry or not (John Wesley's *Journal*, vol. 1, 1737, Friday, 4 March), and in the practices of present-day primitive tribes we get an echo from the classical era. At that time pebbles of diverse shapes and colours, arrows, astragali and dice were all used to probe the divine intention. In the temples there were various and varied rites attached to the process of divination by lot, but the main principle was the same. The question was posed, the lot was cast, the answer of the god was deduced. The dice (astragali, etc.) were thrown sometimes on the ground, sometimes on a consecrated table.

It was customary in classical Greece and Rome for the four astragali of the gamblers to be used in the temples. The prediction was that the throw of Venus (1, 3, 4, 6 uppermost) was favourable and the dogs unfavourable. In the temple of the oracles tablets were hung up and the priest, or possibly the suppliant, interpreted the throw of the four bones by reference to the tablets. Cases have been recorded, however, where five astragali were used. Greek inscriptions found in Asia Minor give a fairly complete record of how the throws of five were interpreted. Each throw was given the name of a god. Thus Sir James Frazer translates (commentary on Pausanias):

> 1. 3. 3. 4. 4 = 15 *The throw of Saviour Zeus*
> One one, two threes, two fours,
> The deed which thou meditatest, go, do it boldly.
> Put thy hand to it. The gods have given these favourable omens.
> Shrink not from it in thy mind. For no evil shall befall thee.

It is not clear whether the order of the numbers is important or not. If order does not matter then the probability of this throw is about 0·08.* The tesserae of the gambler were also used in divination ceremonies as well as the astragali, and it is possible that the same interpretation was given to the numbers falling uppermost, although the presence of the 2 and the 5 would make this a little awkward.

* I propose to write at greater length on 'divination probabilities' on a further occasion.

In addition to the divination carried out by the priests it was apparently a commonplace for individuals to perform acts of divination with regard to events in their daily lives. Thus Lucian, telling the story of the young man who fell madly in love with Praxiteles's Venus of Cnidos, writes:

> He threw four knucklebones on to the table and committed his hopes to the throw. If he threw well, particularly if he obtained the image of the goddess herself, no two showing the same number, he adored the goddess, and was in high hopes of gratifying his passion: if he threw badly, as usually happens, and got an unlucky combination, he called down imprecations on all Cnidos, and was as much overcome by grief as if he had suffered some personal loss.

Again we have from Propertius:

> When I was seeking Venus (i.e. good fortune) with favourable tali, the damned dogs always leaped out.

12. It is perhaps of interest here to interpolate a note on divination as reported practised by the Buddhists of present-day Tibet. According to Hastings (*Dictionary of Comparative Religions*) the simplest method is carried out by the people themselves. Many laymen are equipped with a pocket divination manual (*mô-pe*) and the augury found by casting lots. This lot-casting can either be odds and evens (the random pouring of grain, pebbles or coins from a horn, cup, etc.) or dice on a sacred board or cards on which there are magic signs, or sheets or passages of scriptures drawn from a bowl. The reincarnation prediction is, it is said by Waddell,* usually carried out by a priest. The rebirth chart seen by the writer consists of 56 2 in. squares (8×7). Each square corresponds to a future state. A six-sided die with letters on it is thrown down on to the rebirth chart, and according to the square on which it lands and the letter which falls uppermost so the priest predicts. Waddell, who visited Tibet as a member of a British Mission, obtained one of these charts and a die (*c.* 1893). He remarks: 'The dice (sic!) accompanying my board seems to have been loaded so as to show up the letter Y, which gives a ghostly existence, and thus necessitates the performance of many expensive rites to counteract so undesirable a fate.' Possibly a similar chicanery was practised in Roman times! It would seem a reasonable inference anyway that the mystery and awe which the religious ceremony would lend to the casting of lots for purposes of divination would prevent the thinking person from speculating too deeply about it. Any attempt to try to forecast the result of a throw could undoubtedly be interpreted as an attempt to forecast the action of the deity concerned, and such an act of impiety might be expected to bring ill luck in its train. In addition, as we have noted, a method for such forecasting could not easily be made owing to the imperfections of most of the dice. On the other hand, it is possible that probabilities were known to the priests since the ceremonial dice are well made.

13. Through the Dark Ages the Christian church appears to have carried on guerilla warfare against gaming with knucklebones and dice. The writers of the Renaissance make many references to bishops who write *de aleatoribus* or *contra aleae ludum* during the first fourteen hundred years of the Christian era. It is likely therefore that the bishops wished to get rid of the sortilege as a religious ceremony, and they succeeded to a certain extent in doing this, although divination by lot still survives to-day in the Moravian sect. What the bishops could not do was to stop men playing games of chance. There are several references in early French literature to gaming. The play of Jean Bodel, *Le Jeu de Saint Nicolas*, written *c.* A.D. 1200, has a scene where thieves are gambling in a tavern. They are playing

* L. A. Waddell, *The Buddhism of Tibet*, W. Heffer and Sons Ltd. 1934 (2nd edition).

the dice game of 'Le hasard',* the rules of which were set down clearly by Pierre-Raymond Montmort in his book some five hundred years later. (*Analys sur le jeux d'azard* 1708, p. 113). Bodel's play is interesting for the suggestion that the thieves knew how to manipulate the dice to produce a desired result.

14. With the invention of printing (*c.* 1450) and its rapid development during the latter half of the fifteenth century the references to games of chance become more numerous, but there seems to be no suggestion of the calculation of probabilities. Thus we find in the writing of François Rabelais—a man who might be expected to know the latest in games of chance as played in taverns—the following interesting passage: 'Then they studied the Art of painting or carving; or brought into use the antic play of tables, as Leonicus hath written of it or as our good friend Lascaris playeth at it'† (*Gargantua and Pantagruel*, Urquhart's translation, Book I, Chapter XXIV). *Gargantua and Pantagruel* was issued in sections at intervals between 1532 and 1552. The date of this reference will therefore be not long after 1532.

The Leonicus of the reference is Nicolas Leonicus Tomeus, a professor of Greek and Latin at Padua who was born at Venice in 1456. He was well known for his learning and philosophical bent and acted as tutor to the English Cardinal Pole when as a young man he visited Italy. According to Erasmus he was 'a man equally respectable for the purity of his morals and the profundity of his erudition'. His letters, which have been translated by Cardinal Gasquet, give an interesting picture of the life of an intellectual of that time. He died at Padua in 1531 and his collected works were printed at Basel in 1532. Rabelais is clearly referring to Leonicus's treatise *Sannutus, sive de ludo talario*, a dialogue in the manner of Plato concerning the game of knucklebones (astragali). There is, however, little relevant to the calculus of probability in this work. The discussion turns on references to the game in Roman literature and a description and argument of the value of the various types of throw.‡

A similar type of disquisition was written by Calcagnini about this time but possibly a little later than that of Leonicus. Celio Calcagnini was born at Ferrara in 1479 and died there in 1541. He was a poet, a philosopher and astronomer of repute; his treatise *Quomodo caelum stet, terra moveatur, vel de perenni motu terrae commentatio*, in which he held that the earth moved round the sun, anticipated Galileo Galilei by some years, for Galileo was not born until 1564. The dissertation of Calcagnini entitled *De talorum, tesserarum ac calculorum ludis ex more veterum* is less philosophical in tone than that of Leonicus. It is of interest to probabilists only in that it was an influence over Cardano, who, from his several references, had clearly studied it closely.

* According to the editor, F. J. Warne, of the text of the play, *Le Jeu de Saint Nicolas*, 'hasart' meant the throw of a certain number of points at dice, varying according to the game played. In present-day probability theory the meaning is of course much wider.

† Rabelais actually wrote 'en usage l'anticque jeu des tables ainsi qu'en a escript Leonicus'. Duchat in the commentary on the 1741 edition says 'Ce n'est point *tables* qu'il faut lire ici, comme dans toutes les Editions, mais *tales*'. Presumably Duchat (followed by later commentators) makes this correction because of the work of Leonicus referred to. It is just possible that Rabelais meant what he wrote and that he was referring to the ancient board game (from which the modern game of backgammon developed) in which the 'men' may have been moved by throwing astragali, the counting of the throws being that described by Leonicus.

‡ I have not been able to trace why Lascaris is mentioned by Rabelais. Andre-Jean Lascaris surnamed Rhyndaconus (1445–1535), a Greek scholar born in Phrygia, was Librarian to François I. He rescued many Greek manuscripts from the Turks. Possibly he collected references to gaming in Greek literature much as Leonicus did for Roman?

15. We arrive at the sixteenth century, then, with a well-known humanist Leonicus, and a great astronomer Calcagnini, writing on games of chance with no attempt or reference to the calculation of a probability. (This does not mean of course that some calculations had not been made in a manuscript which we do not know about.) There were, moreover, other scholars and bishops writing on the same topic about this time, so that interest in the subject was keen. As far as we know at present it was left to Gerolamo Cardano to make the step forward. Cardano, the illegitimate son of a geometer, Fazio Cardano, was born in Pavia in 1501. His illegitimacy was a bar to his professional advancement on more than one occasion, and it is possible that the bitterness engendered by this fact was responsible for his not too scrupulous regard for the attribution of other scientists' ideas. The crime of plagiarism was a common accusation among scientific workers of the sixteenth and seventeenth centuries, but of none was it raised more loudly than of Cardano who was strongly disliked by his contemporaries and despised by his successors. Until about the middle of the nineteenth century his biographers unite in regarding him as a charlatan; possibly at the present time the pendulum has swung too far the other way, and more is read into his writings than is justified. The truth would seem to lie somewhere between the extremes of charlatan and persecuted savant.

Cardano was physician, philosopher, engineer, pure and applied mathematician, astrologer, eccentric, liar and gambler, but above all a gambler. He himself owns that on one occasion he sold his furniture and his wife's possessions in order to get money to indulge his passion for gaming, and there is no doubt that this passion was one of the things which ruled him through his whole life. His chief interest professionally was medicine, but he interested himself also in the communication of spirits and the casting of horoscopes. He does not seem to have been too successful at this last, but he was not deterred from casting that of Jesus, a performance the impiety of which probably led to his imprisonment. Even allowing for the exaggerations of his biographers there seems to be no doubt that he was eccentric to the point of madness. This did not prevent him, however, from making contributions to pure mathematics, and it is to this combination of pure mathematician and gambler that we owe the *Liber de Ludo Aleae*. This treatise was found in manuscript in Cardano's papers after his death in Rome in 1576, and was first published in his collected works in 1663 at Lyon. Cardano implies that it was written *c.* 1526; the exact date is not important since no question of priority or plagiarism is involved, but it is curious that a manuscript of this kind should have survived fifty years of his remarkably variegated career.

16. The first complete translation of *de Ludo Aleae* into English is given in *Cardano, the Gambling Scholar*, by Oystein Ore, published in 1953. Ore remarks that the book is badly composed and that understanding of Cardano's work has possibly been hindered by this. There are some, however, who will not agree with his commentary on the treatise and who may feel that as much prescience is now attributed to Cardano as there was before too little. The crux of Cardano's work is to be found in the section entitled 'On the cast of one die' in Ore's translation:

> The talus has four faces and thus also four points. But the die has six; in six casts each point should turn up once; but since some will be repeated, it follows that others will not turn up. The talus is represented as having flat surfaces, on each one of which it lies on its back;...and it does not have the form of a die. One half of the total number of faces always represents equality; thus the chances are equal that a given point will turn up in three throws, for the total circuit is completed

in six, or again that one of three given points will turn up in one throw. For example, I can as easily throw one, three or five as two, four or six. The wagers are therefore laid in accordance with this equality if the die is honest....

We have therefore the necessary abstraction made; if the die is honest, i.e. if we may give equal weight to each side, then we may calculate the chances. There is no doubt, I think, that Cardano was led to this conclusion empirically and his generalization of it is partially wrong. For he goes on to discuss casts of two dice and three dice giving tables which are correct if 'the dice be honest'. When we come, however, to the section 'On play with knucklebones', it seems that he falls into error. The knucklebones (or astragali) have four sides. The different combinations of numbers which may arise in the throwing of four astragali are correctly enumerated, but the chances are calculated under the assumption that all sides are equi-probable; which they are not. Possibly Cardano had never played with astragali, for it is likely that if he had he would have noticed that to assume the sides of the astragalus had equal weight in his enumeration of alternatives was not adequate. But this fumbling suggests that he was not quite clear in his own mind about what he was proposing.

I do not think that the fact that Cardano did not quite see the mathematical abstraction clearly can detract from the fact that he did, on paper at any rate, as far as we know, calculate the first probability by theoretical argument, and in so doing he is the real begettor of modern probability theory. The claims of his biographer that he anticipated the law of large numbers, etc., may not be acceptable; it would appear that Cardano was judging from his experience rather than his algebra.

17. It would be strange if Cardano, following the mode of his age, did not communicate some of his thoughts about gaming to his pupils. Fear of being accused of plagiarism, fear of being plagiarized, may have kept him silent, but the whole tone of his treatise is a practical one; practical advice about playing, laying odds and so on make up a large portion of it. He would therefore almost certainly have discussed its contents with his friends, particularly if he thought about it over as long a period of time as he suggests. The fact that *de Ludo Aleae* did not appear in print until 1663 does not therefore seem to be a reason why Cardano's ideas should not have been common knowledge to scholars in Italy after his death, and the way in which Galileo-Galilei plunges into *his* discussion of dice playing, without much preamble, tends to lend colour to this.

Galileo-Galilei was born in Pisa in 1564, the son of Vincent Galilei, a musicographer well known in his day. He died in 1642 at Arcetri after a career as full of achievement as any that has ever been known. His contributions to science, both as astronomer and as mathematician, are striking for their originality of thought and clarity of purpose. Why this prince of scholars has never received the full recognition which is his due it is difficult to say. It is thought by some modern writers that his sensible recantation of the earth's movement, after physical torture at the hands of the Inquisition at the age of 70, has caused a revulsion to him among the scientists of later years. This is probably not so; what is more likely is that the envious fellow-scholars who delivered him to the Inquisition conspired after his death to belittle the work which he had done. In this they were possibly helped by Galileo's literary style which is noteworthy for clarity but not brevity, being in fact prolix and tedious in the extreme; no *i* is left undotted, no *t* is left uncrossed.*

* E. S. Pearson suggests to me that this prolixity was one which Galileo shared with many other Renaissance writers, and that it arose from the struggle which the early mathematicians must have had to formulate mathematical abstractions on paper. I think that this may well be so.

This being so if there was any doubt about the general method of procedure in calculating chances with a die we should have had a long disquisition on the subject. However, in *Sopra le Scoperte de i Dadi** he plunges straight away into his argument. The problem† is one already touched on by Cardano. Three dice are thrown. Although there are the same number of three partitions of 9 as there are of 10, yet the probability of achieving 9 in practice is less than that of throwing 10. Why is this? I quote a little from E. H. Thorne's translation of this note. The note begins:

> The fact that in a dice game certain numbers are more advantageous than others has a very obvious reason, i.e. that some are more easily and more frequently made than others, which depends on their being able to be made up with more variety of numbers. Thus a 3 and an 18, which are throws which can only be made in one way with 3 numbers (that is, the latter with 6, 6, 6 and the former with 1, 1, 1, and in no other way) are more difficult to make than, for example, 6 or 7, which can be made up in several ways, that is a 6 with 1, 2, 3 and with 2, 2, 2 and with 1, 1, 4 and a 7 with 1, 1, 5; 1, 2, 4; 1, 3, 3; 2, 2, 3. Again, although 9 and 12 can be made up in as many ways as 10 and 11 and therefore they are usually considered as being of equal utility to these, nevertheless it is known that long observation has made dice players consider 10 and 11 to be more advantageous than 9 and 12.

This extract serves to show how he begins the topic assuming that the calculations are known; it also serves to illustrate the prolixity of Galileo's style. After some discussion of the six 3 partitions of 9 and of 10, he goes on:

> Since a die has six faces and when thrown it can equally well fall on any one of these, only six throws can be made with it, each different from all the others. But if together with the first die we threw a second, which has also six faces, we can make 36 throws each different from all the others, since each face of the first die can be combined with each of the second....

After saying that the total number of possible throws with three dice are 216, he gives a table of the number of possible throws for a total of 10, 9, 8, 7, 6, 5, 4, 3, noting that the numbers 11–18 inclusive are symmetrical with these. Thus the number of possible throws for 10 is 27 and 25 for 9. His treatment of the problem is exactly that which we should use to-day and leaves us in no doubt that the calculation of a probability from the mathematical concept of the equi-probable sides of the die was clearly known to the sixteenth-century mathematicians of Italy. We can marvel at the person asking Galileo the question; he obviously gambled sufficiently to be able to detect a difference in empirical probabilities of 1/108.‡

18. Galileo's collected works were first published in Bologna in 1656, but this fragment on gambling was not included. It does appear in the more complete collection published at Florence in 1718. Since, however, Galileo thought the problem of little interest, for he did not pursue it, there seems to be no reason why he should have made a secret of it, and following the custom of his day he probably instructed his pupils. At any rate it is evident that the mathematical probability set was no stranger to the French mathematicians of the seventeenth century, as is witnessed by the now famous correspondence between Pascal and Fermat in 1654. The first letter of the series, from Pascal to Fermat, setting out the problem of points is missing. We have, however, Fermat's reply to it, and the subsequent

* This is Galileo's own title. *Considerazione sopra il Giuco dei Dadi*, a later title, appears first in the collected works of 1718.

† Like Pascal sometime later, Galileo wrote to answer a problem put to him by a gambler.

‡ M. G. Kendall points out to me that the problem posed by the Chevalier de Méré to Pascal concerning the problem of points involved similar small probabilities.

follow-up,* and from the way in which Fermat writes it seems clear that the actual definition of probability is assumed known. What the two savants were interested in was the application of this definition to specific problems which were concerned with dice playing between gamblers of equal skill and opportunity. The approach to the problems is similar to that of Galileo, and the generalizations which are made from the particular cases discussed are not well supported.

It is true that Galileo wrote on one problem only and fairly briefly at that, but it is difficult to see why Pascal and Fermat should be preferred as the originators of probability theory before Galileo or Cardano. It may well be that the precocity of Pascal as a mathematician led to much of his work being accepted with acclamation, and certainly without its priority being questioned. We find, for example, the famous Arithmetic Triangle in Stifel's *Arithmetic* (1543), in the *General Tratato* of Tartaglia in 1556, in the *Arithmetic of Simon Steven of Bruges* (Leiden, 1625). It is possible that Pascal may not have known of these writers. However, he certainly knew of Pierre Herigone's *Cours Mathematique* (Paris, 1634), since he makes several references to it in his own *Usage du Triangle Arithmétique pour trouver les puissances des binômes et Apotômes*. Herigone uses a table of numbers analogous to the Arithmetic Triangle to find binomal coefficients. Perhaps this same aura which dazzled Pascal's contemporaries (and at the same time caused them to overlook some of Fermat's work) still blinds us to-day.

19. If we take the origins for granted and look at developments of the theory, then by far the greatest impetus to theory during the years 1650–60 must have come from the publication of *De Ratiociniis in Aleae Ludo* by Christian Huygens. Huygens as a young man of 26 arrived in Paris in July 1655 on the equivalent of the English 'Grand Tour'. He did not meet Pascal, Fermat or Carcavi, the intimate friend of Pascal, but he did meet Roberval, professor of mathematics at the Collège Royal de France, who is mentioned by Pascal as having been also approached by the Chevalier de Méré. Huygens stayed in Paris from July to November, and after his return to Holland he began a correspondence with both Carcavi and Fermat which lasted over a period of years. The young man's imagination was obviously fired by the discussions he had in Paris, and his mathematical ambitions stimulated by the immense activity of the group which some ten years later (1665) was to found the *Académie des Sciences*. He set himself to work, and in March 1656 he wrote to Prof. van Schooten that he had prepared a manuscript about dice games. Francis Schooten was professor of mathematics at Leyden and had been Huygens's teacher. He took the young Huygens's manuscript (which was written in his native language), translated it into Latin and published it as an appendix to his *Exercitationes Mathematicae* in 1657. (A French translation of this appendix can be found in *Oeuvres de Huygens*, tome 14, on 'Calcul des Probabilités' published by *La Société Hollandaise des Sciences* in 1920.) In this *Tractatus de Ratiociniis in Aleae Ludo* Huygens sets out in a systematic manner what he must have learnt in Paris and adds some results which he may have achieved himself.

In the letter to Francis Schooten he writes

> ...quelques-uns des plus Célèbres Mathématiciens de toute la France se sont occupés de ce genre de Calcul, afin que personne ne m'attribue l'honneur de la première Invention qui ne m'appartient

* It is interesting to see Pascal fall into the same kind of trap which caused D'Alembert such controversy. In discussing the game of heads and tails and the tossing of a coin D'Alembert argued that the probability of throwing a head with two tosses of a coin was 2/3. For we may have *TT*, *TH* or *H*—when we stop, the second throw being immaterial, since we have achieved what we want.

pas. Mais ces savants...ont cependant cachés leurs méthodes. J'ai donc dû examiner et approfondir moi-même toute cette matière à commencer par les éléments, et il m'est impossible pour la raison que je viens de mentionner d'affirmer que nous sommes partis d'un même premier principe....

Accordingly Huygens begins by proving his basic propositions, deals at some length with the problem of points and then passes on to dice playing. His last proposition (XIV) has a familiar ring:

> If another gambler and I throw 2 dice turn and turn about with the condition that I will have won when I throw 7 points and he will have won when he throws 6, if I allow him to throw first, find my chance and his of winning.

His delineation of his fourteen propositions is admirably clear and concise, and it is no marvel that the tract was used by mathematicians as a reference book up to the time of James Bernoulli (who reprinted it) and beyond. Possibly by this crystallization of the ideas of the French mathematicians Huygens has earned the right to be regarded as the father of the probability theory.

20. After Huygens the interest of probabilists was not solely in gaming, although this interest did not die away for another hundred years or so. But with Huygens the new calculus seems fairly launched, and this is therefore a suitable point to make a break. There are many questions which one leaves unanswered. The drawings and paintings by palaeolithic man of himself are very rare, and there is probably no hope of finding pictures of his recreations. If he prized the astragalus as a toy it seemed a possibility that he might have carved or decorated it in some way, but I have not been able to find any record of this. But while we cannot pull aside the curtain from four hundred centuries the possibility does exist that the pre-historians may be able, one day, to take us back a little farther than the third millennium. The farther back one goes the more fragmentary the evidence, but the earliest dice found are described as being of 'well-fired buff pottery', and they certainly would not have been the first made.

The tantalizing period to the present writer is the period from the invention of printing to A.D. 1600. In this period we have two mathematicians only calculating probabilities, and yet this was in the immense intellectual ferment of the Italian Renaissance. It seems hardly possible that there were not other natural philosophers who attempted similar calculations, but such documents, if they exist, will only now come to light by chance.

The correspondence between the French mathematicians of the first half of the seventeenth century is almost complete, and presumably the possibility does exist here of finding further letters. They all seemed at one time or another to send letters to one friend under cover of letters to another, and such letters may conceivably still be ascribed to the wrong person. However, enough information does exist regarding the seventeenth-century mathematicians to make a coherent study, and if I appear to have done them scant justice it is because I find the period so interesting that I hope to write about it more fully on another occasion elsewhere.

Collecting information about dicing and gaming has been a hobby of mine for some time, and the list of persons who have drawn my attention to one aspect or another of it is formidable. I want to thank Prof. B. Ashmole of the British Museum who allowed me critically to examine the dice of the classical period which are in his care and M. Jean Charbonneaux of the Musée de Louvre who did me the same service. To Prof. C. M. Robertson of my own college I owe not only the privilege of tossing many dice but many stimulating discussions and useful references. A. J. Arkell allowed me to examine the dice brought by Prof. Sir Flinders Petrie from Egypt and to photograph various gaming boards not reproduced here. The breadth of knowledge and wide reading of Miss M. S. Drower have acquainted me with many Egyptian board games which provide a fascinating puzzle for those interested in deducing how they are played. Miss J. Lowe and R. Graves drew my attention to various references in classical literature. The illustration of the Hounds and Jackals game is by the courtesy of the Metropolitan Museum of Art of New York.

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